§ 2.4: Transformations of Functions

Vertical Shifting

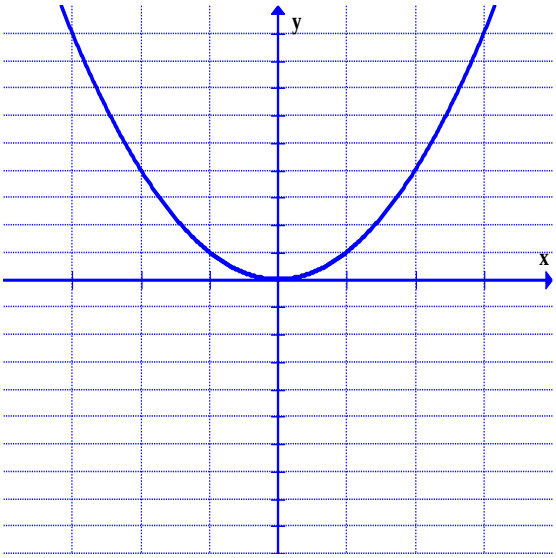
Adding a constant to a function shifts its graph vertically: upward if the constant is positive and downward if the constant is negative.

Example 1	Vertical Shifts of Graphs

Use the graph of $f(x) = x^2$ to sketch the graph of each function.

(a) $f(x) = x^2 + 2$

(b) $f(x) = x^2 - 3$



Vertical Shifts of Graphs

Suppose c > 0.

To graph $y = f(x) + c$, shift the graph of $y = f(x)$ upward c units.

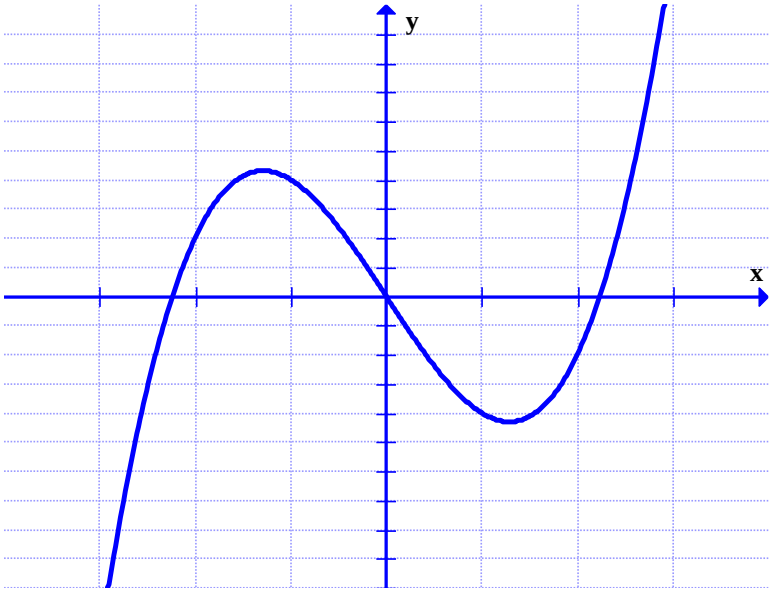
To graph $y = f(x) - c$, shift the graph of $y = f(x)$ downward c units.

Example 2	Vertical Shifts of Graphs

Use the graph of $f(x) = x^3 - 5x$, which is sketched below, to sketch the graph of each of the following.

(a) $f(x) = x^3 - 5x + 3$

(b) $f(x) = x^3 - 5x - 2$



Horizontal Shifting

Suppose we have the graph of $y = f(x)$, how would we use this graph to sketch the graphs of $y = f(x + c)$ and $y = f(x - c)$, where c > 0?

The y value of $f(x - c)$ is the same as $f(x)$ evaluated at $x - c$. Since $x - c$ is c units to the left of x, it follows that the graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted to the **RIGHT** c units.

Horizontal Shifts of Graphs

Suppose c > 0.

To graph $y = f(x - c)$, shift the graph of $y = f(x)$ to the right c units.

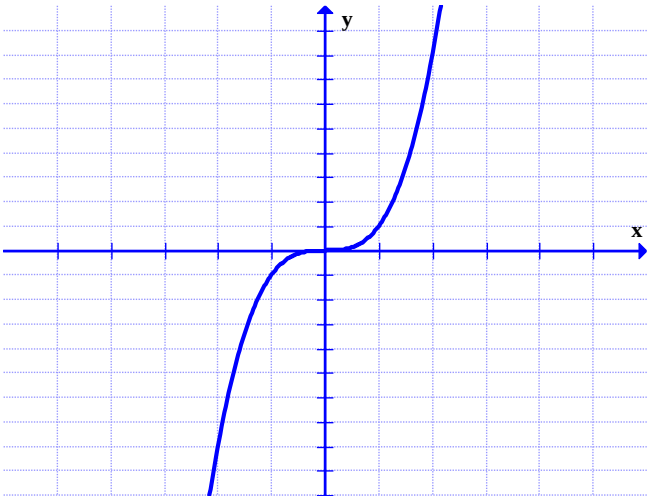
To graph $y = f(x + c)$, shift the graph of $y = f(x)$ to the left c units.

Example 3	Horizontal Shifts of Graphs

Use the graph of $f(x) = x^3$ to sketch the graph of each function.

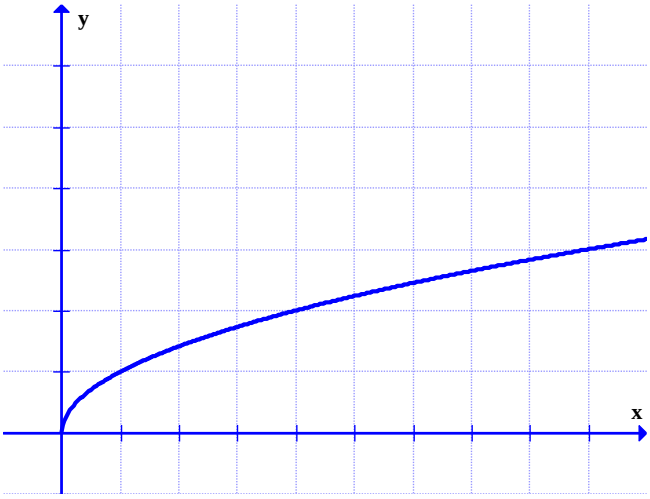
(a) $f(x) = (x-3)^3$

(b) $f(x) = (x+2)^3$



Example 4	Combining Horizontal and Vertical Shifts

Sketch the graph of $f(x) = \sqrt{x-4} + 2$, starting with the graph of $f(x) = \sqrt{x}$ shown below



Reflecting Graphs

Suppose we have the graph of the function $y = f(x)$, how would we use this graph to find the graphs of $y = -f(x)$ and $y = f(-x)$?

Notice that the y value of $y = -f(x)$ is simply the opposite of the y value of $y = f(x)$, and therefore the graph of $y = -f(x)$ is simply the graph of $y = f(x)$ reflected about the x-axis.

On the other hand, the value of $y = f(-x)$ is the same as $y = f(x)$ evaluated at $-x$, and therefore the graph of $y = f(-x)$ is simply the graph of $y = f(x)$ reflected about the y-axis.

Reflecting Graphs

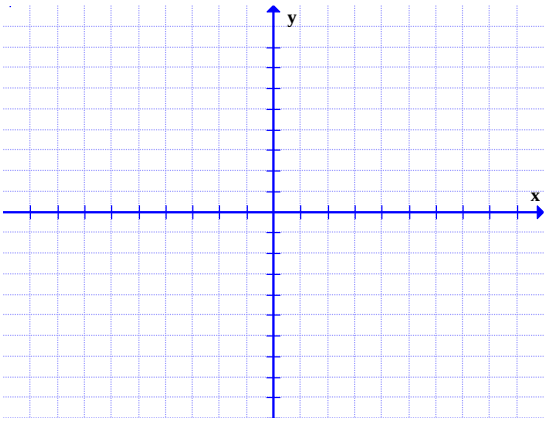
To graph $y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis.

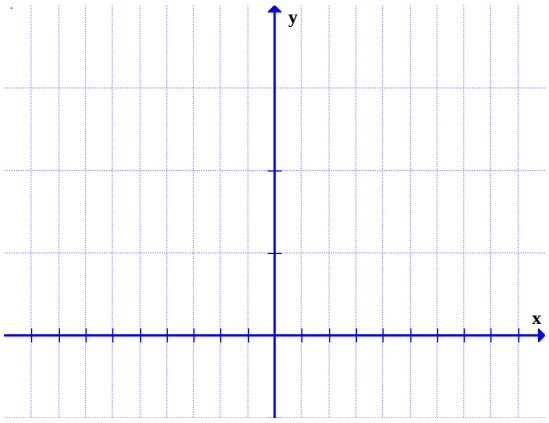
To graph $y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis.

Example 5	Reflecting Graphs

Sketch the graph of each function

(a) $f(x) = -x^2$ (b) $g(x) = \sqrt{-x}$





Vertical Stretching and Shrinking

Suppose we have the graph of $y = f(x)$. How would we use this graph to sketch the graph of the function $y = cf(x)$?

Note that the *y*-coordinate of $y = cf(x)$ is equal to *c* times the *y*-coordinate of $y = f(x)$. Multiplying the *y*-coordinates by the factor of *c* has the effect of vertically stretching (c>1) or shrinking (0<c<1) the graph by a factor of *c*.

Vertical Stretching and Shrinking of Graphs

To graph $y = cf(x)$:

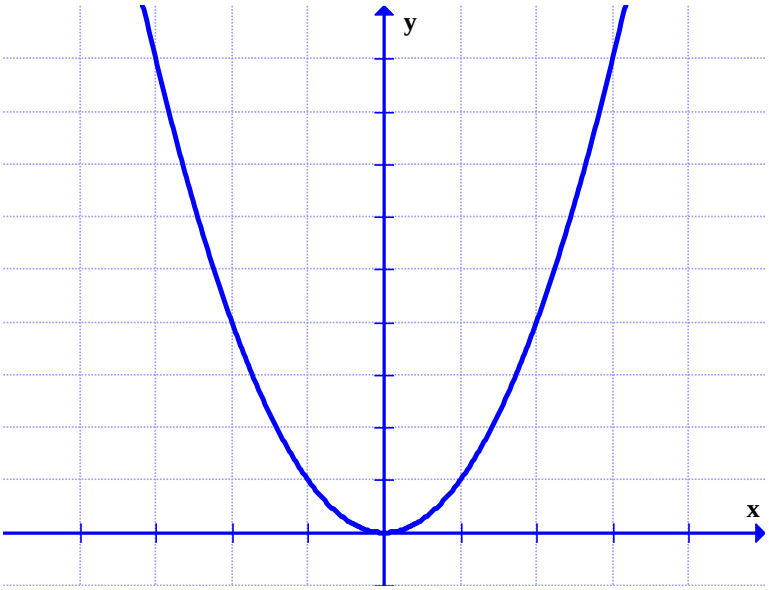
If c > 1, then stretch the graph of $y = f(x)$ vertically by a factor of *c*.

If 0 < c < 1, then shrink the graph of $y = f(x)$ vertically by a factor of c.

Example 6	Vertical Stretching and Shrinking of Graphs

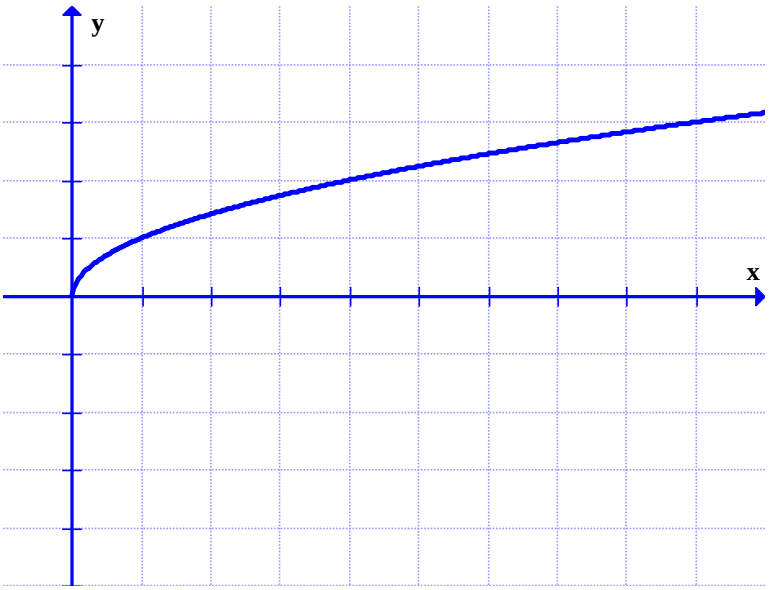
Use the graph of $f(x) = x^2$ to sketch the graph of each of the following.

(a) $f(x) = 3x^2$ (b) $f(x) = \frac{1}{2}x^2$



Example 7	Combining Shifting, Stretching, and Reflecting

Sketch the graph of $f(x) = 1 - 2\sqrt{x-3}$.



Horizontal Stretching and Shrinking

Horizontal Shrinking and Stretching of Graphs

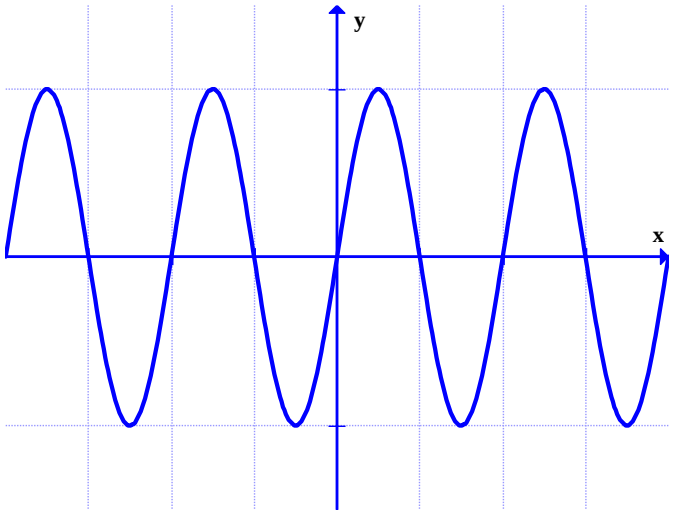
To graph $y = f(cx)$

If c > 1, then shrink the graph of $y = f(x)$ by a factor of $1/c$.

If 0 < c < 1, then stretch the graph of $y = f(x)$ by a factor of $1/c$.

Example 8	Horizontal Stretching and Shrinking of Graphs

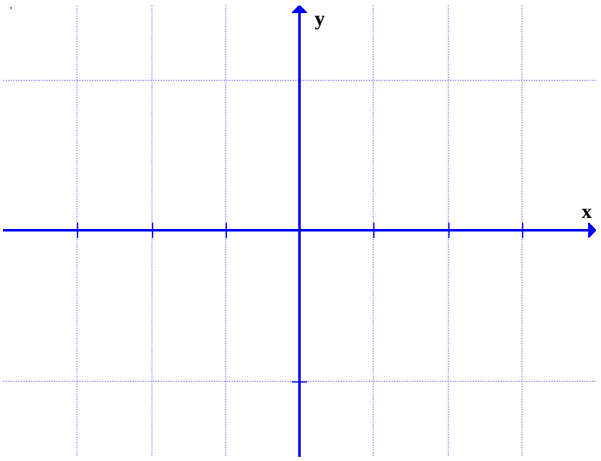
The graph of the function $f(x) = \sin(\pi x)$ is graphed below.

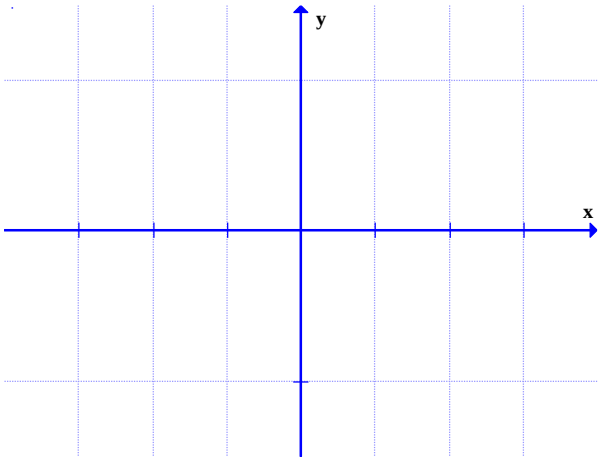


Use this graph to graph

(a) $f(x) = \sin(2\pi x)$

(b) $f(x) = \sin\left(\frac{\pi x}{2}\right)$





Even and Odd Functions

Even and Odd Functions

Let f be a function.

f is **even** if $f(-x) = f(x)$ for all x in the domain of f.

f is **odd** if $f(-x) = -f(x)$ for all x in the domain of f.

Example 9	Even and Odd Functions

Determine whether the functions are even, odd, or neither even nor odd.

(a) $f(x) = x^2 + 5$

(b) $g(x) = x^7 + x^5 + x$

(c) $h(x) = x^2 + x$

Homework

Due:_____

2 – 16 (even), 20, 28 – 32 (even), 34 – 48 (even)